THE AMERICAN MATHEMATICAL MONTHLY

VOLUME XX APRIL, 1913 NUMBER 4

SOME THINGS WE WISH TO KNOW

1. **Dangers.** It may be well to mention first some of the things we do not wish to know. Broadly speaking, these fall under two heads: (*a*) purely personal opinions not supported by facts or experiments; (*b*) exhaustive treatments of the entire field.

Topics that are too broad in their scope are attractive to the writer on pedagogical material because they admit of broad generalities and have the allurement of apparently larger accomplishment. This is just the reason why we do not care to encourage them, for it is evident that a broad topic, such as *The value of mathematics,* or *How algebra should be taught,* not only allows glittering generalities of statement, but is likely to contain very little of specific worth on any one given point.

Broad topics also savor strongly of the kind of *ex cathedra* statements of personal opinion mentioned above. A statement that a given topic is (or is not) vital, or interesting, or valuable, or good for the student, without supporting reasons or statistics, is very common, especially in articles on mathematics. This practice we cannot too strongly condemn and we intend to discourage it to the limit of our ability; not that we would contend that a given topic is *not* interesting, for on the contrary, we are inclined to believe that a great variety of topics are extremely interesting. But that is not getting very far in deciding whether the topic should be taught, and how.

2. **The Basis of Knowledge.** It seems to demand an apology if we state to readers who are scientists that the basis of knowledge not received by revelation is either experimentation or deduction. As a defense for such a truism, we need only point to the many papers that show evidence of some other method of reaching conclusions.

In the Dark Ages, and for a period thereafter, medical practice included the administration of blood-letting; and it was often asserted that this process was beneficial to the patient. They who first arose to demand statistics upon this and other hallowed practices were naturally regarded as heretics, and they were treated with due scorn and derision.

May we very hesitatingly ask for a few statistics regarding some of our own hallowed traditions? And do we thereby incur displeasure? Shall it be said of us that we are attempting the overthrow of mathematics? In any event, that is what we mean by *knowing*, for we are aware of no other basis of knowledge than such experimentation as we here imply.

3. **Several Topics.** Among fair topics for such a really valuable discussion, which we would hold to be quite open to both opinions, the usual process for discovering the rational roots of an ordinary algebraic equation seems to be a good illustration.

In how far is this topic a desirable one? We know that it is theoretically always effective. But in how far is it really called for? In how far do students that have been taught the process retain it? How long do they retain it? Do they actually use it? Is the difference between groups of students (not individuals) that have been taught the process and those who have not, traceable in any manner after one year? What difference, if any, manifests itself, in such groups, either in knowledge or in behavior?

The consideration of the possible value of having learned, even a forgotten topic, is here by no means ruled out of consideration. How does this value manifest itself? If it is real, it should be traceable in a *group*. But is this the *only* claim which is valid for this particular topic?

These things we wish to know. They can be found out. Nor is the topic too small to warrant a critical examination and report. We most earnestly assert that the records of real work on this topic would be more welcome and more valuable than an extensive expression of mere opinions on the whole of algebra.

The amount of work necessary to really contribute to such a topic becomes apparent upon short consideration. There are other things we wish to know that can be settled quickly. For example, we would be glad to know what percentage of men entering various universities know how to solve a numerical quadratic; what percentage know how to solve a linear equation; what percentage know how to add, subtract, multiply and divide with simple fractions such as 3/5 and 4/7. Here again mere expressions of opinion as to what should be the case are not desired. We would be glad to know how these percentages change every six months for two years. Does the training received in mathematical courses perceptibly affect the ability to manipulate simple fractions? Safeguards against erroneous conclusions are to be found in books and papers on psychology.

4. **A Question about Tables.** Another question that is in serious need of discussion is the most efficient use of types and spacings in tabular matter of all sorts. The variety of types in actual use suggests the desirability of some accurate study, and the mention of psychology suggests it. To be sure, experiments have already been made. These, though valuable, are not complete nor exhaustive. What has been done is quite unknown, apparently, among mathematicians. Here again, expression of personal preference is not at all trustworthy or valuable. Nor is the persistence of an antiquated form of type in tabular matter an argument in its favor, as has been pointed out frequently in reference to the German

"Gothic" characters. That the spacing into groups of threes or into groups of fives laterally and vertically has an important bearing upon legibility and quick understanding is highly probable; that the use of white spaces in the place of black rulings works great change is undoubted. Which of these, and what other features are to be preferred in tables, is of great consequence. These questions can only be answered after very exhaustive experiments of no mean order. A competent psychologist and a competent mathematician working jointly would make the best combination for attempting such a great task.

5. **Forthcoming Articles.** We are glad to announce that short articles dealing with very specific topics are already promised. Among others we may mention: an article on the teaching of solid angles; one on the desirability of accurate space drawings; one on methods of interpolation in a table of logarithms of the trigonometric functions for small angles; one on the elements of the theory of insurance. It is hoped that these and others equally definite in topic will not be long delayed. In this connection see the note at the bottom of page 140 of this issue.

E. R. Hedrick, *for the editors.*

HISTORY OF THE EXPONENTIAL AND LOGARITHMIC CONCEPTS.

By FLORIAN CAJORI, Colorado College.

IV. From Euler to Wessel and Argand. 1749–about 1800.

A Half Century of Barren Discussion on Logarithms.

In an essay *Sur les logarithmes des quantités negatives*[1] D'Alembert refers to his correspondence of 1747 and 1748 with Euler on this subject and states that he had read Euler's paper of 1749, but felt still that the question was not settled. D'Alembert proceeds to advance arguments of metaphysical, analytical and geometrical nature which shrouded the subject into denser haze and helped to prolong the controversy to the end of the century. Defining logarithms by the aid of two progressions, as did Napier, D'Alembert asserts that the logarithms of negative numbers are not imaginary, but real, or rather, that they may be represented at will as real or imaginary, since everything hinges on the choice of the system of logarithms. As his metaphysical argument against Euler's conclusion he gives it as improbable that the logarithmic curve $y = e^x$ passes from $x = \infty$ to imaginary values. As a geometric reason he asserts that all curves for which $a^n dy/y^n = dx$, n being odd, have two branches which are symmetrical with respect to the X-axis and yielding two values, y and $-y$ for one and the same value of x. He claims his proof to be general, hence true for $n = 1$.

[1] *Opuscules mathématiques*, T. I, Paris, 1761, pp. 180–209.